Predicting residual stresses in additive manufacturing through the application of inherent strain method

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December 15, 2024

1 Introduction

In this post, we introduce the basic formulation for predicting the residual stresses during the process of SLM (selective laser melting) additive manufacturing. In SLM, the component is manufactured by vertical deposition and selective melting of thin layers of metal powder (typically of the order of 40-100μm) by a focused heat source in the form of a laser beam. High thermal gradients inherent to this process result in the formation of residual stresses and distortion which may have significant contribution on the overall quality of the printed part as well as success of the printing process (which may involve part cracking and excess distortions damaging the recoater).

We will illustrate the concepts on a simple cuboidal geometry illustrated in Fig.1.

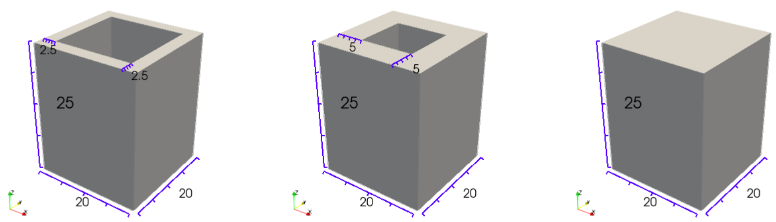

Figure 1: Simple cuboidal structure with varying wall thickness.

2 Objective

Predict distortion and residual stresses in SLM using the inherent strain method.

3 Governing equations

Stress distribution in a mechanically equilibrated solid can be found by solving the following equation

$$\nabla \cdot \boldsymbol{\sigma} = \mathbf{0} \tag{1}$$

For simplicity, in this example, we will neglect the effects associated with plastic deformation, and consider a simple linear-elastic material model. Nevertheless, the plasticity can be easily accommodated by modifying the constitutive model, while the remainder of the framework remains the same. In such case, the stress is linearly proportional to the elastic strain.

$$\boldsymbol{\sigma} = \mathbb{C} : \boldsymbol{\varepsilon}^e \tag{2}$$

The total strain can be additively decomposed into various components, such as

$$\boldsymbol{\varepsilon} = \boldsymbol{\varepsilon}^e + \boldsymbol{\varepsilon}^i \tag{3}$$

where $\boldsymbol{\varepsilon}^e$ is the elastic component of strain tensor, $\boldsymbol{\varepsilon}^i$ represents the inherent strain (associated with cooling process and thermal strains, which are not modelled explicitly). Other strain components such as those associated with plasticity or solid-solid phase transformation can be added.

Assuming the linearised measure of strain tensor

$$\boldsymbol{\varepsilon} = \frac{1}{2}\left(\nabla \boldsymbol{u} + (\nabla \boldsymbol{u})^T\right) \tag{4}$$

While we expect the overall deformations to be relatively small (which would justify the choice of linearized formulation), finite strain theory may be necessary for predicting more exotic behaviour such as buckling in thin wall structures (see for instance [**?**]).

The isotropic tensor of elastic constants $\mathbb{C}$ can be expressed in terms two material parameteres, for instance the Lame parameters λ and μ, which can be then written in the following form:

$$\mathbb{C} = \lambda \boldsymbol{I} \otimes \boldsymbol{I} + 2\mu \mathbb{I} \tag{5}$$

Up to this step it be seem that we are dealing with a linear problem. However, the non-linearity enters in the form of layer deposition where the current deformation needs to be accounted for. This is illustrated in Fig.2 where, in order to predict the build-up of residual stresses, deformation of the system needs to be considered before addition of a new layer of material. For a given layer heigh h, the volume of newly deposited material (number of powder particles) will depend on the current deformation of the consolidated part. Notice, in Fig.2 how the number of particles changes as the build (and deformation of the top surface) progresses.

To resolve this, that is to account for the constant layer height, various approaches can be applied. Here, we propose to include another strain component $\boldsymbol{\varepsilon}^*$ accounting for the current deformation of the system. The total strain tensor will be therefore decomposed into

$$\boldsymbol{\varepsilon} = \boldsymbol{\varepsilon}^e + \boldsymbol{\varepsilon}^i + \boldsymbol{\varepsilon}^* \tag{6}$$

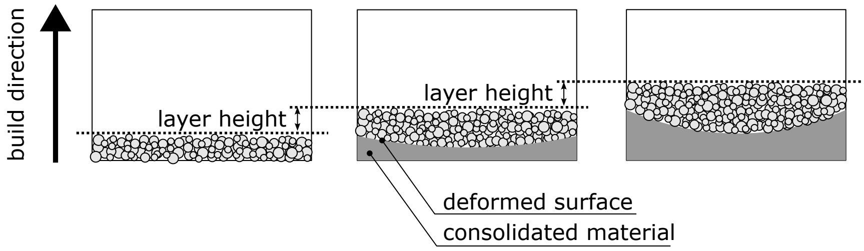

Figure 2: Exaggerated illustration of the SLM process during which the consolidated layer of powder deforms as it cools down. Therefore, for a given layer heigh h, the volume of newly deposited material will depend on the current deformation of the consolidated part.

The shape (and volume) of this new layer will depend on the current overall deformation of the system. Therefore, the algorithm of simulating this process must be divided into two steps. In the first step, the linear problem is solved where the distortion and residual stresses are predicted under the application of the inherent strain. In the second step, the geometry is updated by adding new volume of material considering the deformed surface at the bottom and flat surface on the top (see Fig.2). By repetition of these two steps, the process is repeated until the desired number of layers.

Below we write the necessary numerical implementation through the finite element method incorporating the aforementioned process.

4 Numerical implementation - finite element model

We will consider the finite element method for solving this problem. This is beneficial especially when solving on the domains of complex geometries that are typically encountered in additive manufacturing.

The domain of the component is divided into a finite number of elements, as illustrated in Fig.??. This mesh was generated using Gmsh (`https://gmsh.info/`).

It is important to note that the number of finite element layers is much less than the number of real layers, where resolving individual layers (of the order of tens of micrometers would be computational intractable. Typically, a sensitivity analysis is performed in order to assess the error of lumping multiple real layers into one finite element layer.

Furthermore, in this example, the scan strategy is not resolved explicitly, while it is assumed that the inherent strain $\boldsymbol{\varepsilon}^i$ is applied uniformly into each newly deposited layer. This parameter, which is typically determined from experiments, implicitly contains all information related to the scan strategy.

The finite element model is established on the assumption of mechanical equilibrium expressed through the equivalence of internal and external forces

$$\boldsymbol{f}^{int} = \boldsymbol{f}^{ext} \tag{7}$$

Since in this case we do not apply any external forces, we end up having to solve

$$\boldsymbol{f}^{int}(\boldsymbol{u}) = \boldsymbol{0} \tag{8}$$

which is a non-linear function of the displacement field.

The internal forces are calculated by integrating stress

$$\boldsymbol{f}^{int} = \sum_i A_i \int_{\Omega_i} \boldsymbol{B}^T \boldsymbol{\sigma} d\Omega_i = \sum_i A_i \int_{\Omega_i} \boldsymbol{B}^T \boldsymbol{C} \left(\boldsymbol{B}\boldsymbol{u} - \boldsymbol{N}\boldsymbol{\varepsilon}^i - \boldsymbol{N}\boldsymbol{\varepsilon}^* \right) d\Omega_i = \boldsymbol{0} \tag{9}$$

Here, $\boldsymbol{B}$ is the strain matrix, and $\boldsymbol{N}$ is the matrix of shape functions, as is standard in the finite element method. For brevity, we will omit further details of the finite element method and refer the reader to more comprehensive literature, for instance [?].

Rearranging, we solve the following linear system of equations

$$\sum_i A_i \left[\int_{\Omega_i} \boldsymbol{B}^T \boldsymbol{C} \boldsymbol{B} d\Omega_i \right] \boldsymbol{u} = \sum_i A_i \int_{\Omega_i} \boldsymbol{B}^T \boldsymbol{C} \boldsymbol{N} \left(\boldsymbol{\varepsilon}^i + \boldsymbol{\varepsilon}^* \right) d\Omega_i \tag{10}$$

which is can be written into a more familiar format

$$\boldsymbol{K}\boldsymbol{u} = \boldsymbol{F} \tag{11}$$

Finally, we define the $\boldsymbol{\varepsilon}^*$ strain

$$\boldsymbol{\varepsilon}^* = \boldsymbol{B}\delta_{n-1} \tag{12}$$

where $\boldsymbol{B}$ is the strain matrix as used before, and δ_{n-1} is the displacement vector

$$\delta_{n-1} = \begin{bmatrix} \boldsymbol{u}_1 & \boldsymbol{u}_2 & \boldsymbol{u}_3 & \boldsymbol{u}_4 & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \end{bmatrix}^T \tag{13}$$

In other words, the application of this strain will pre-deform the finite element to a shape conformal to the deformed surface of the part. This is illustrated in Fig.3.

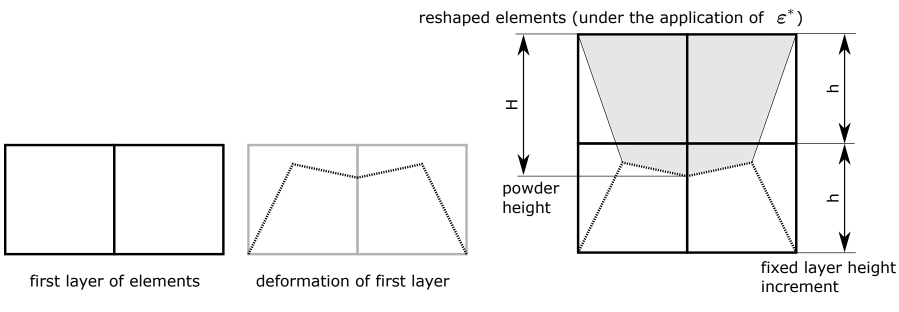

Figure 3: Implementation of the overall method at the level of individual finite elements.

Initially undeformed finite elements undergo the transformation due to the application of inherent strain. The system deforms as schematically shown in

Fig.3 (middle). A new layer of finite elements is added with element shapes dictated by ε^i such as to fit the bottom deformed surface and having a flat surface at the top (c.f. Fig.2).

5 Simulations

In this, simulation results of the building process are presented for the three structures (depicted in Fig.1).

6 Conclusion

In this post, we have illustrated a simple finite element model for predicting residual stresses in SLM additive manufacturing. The model can be applied to arbitrarily complex geometry.

7 Problems

7.1 Problem 1

Using this approach (or a similar one you have developed), investigate which configuration A or B presented in Fig.4 will lead to a component with smaller distortions or smaller residual stresses.

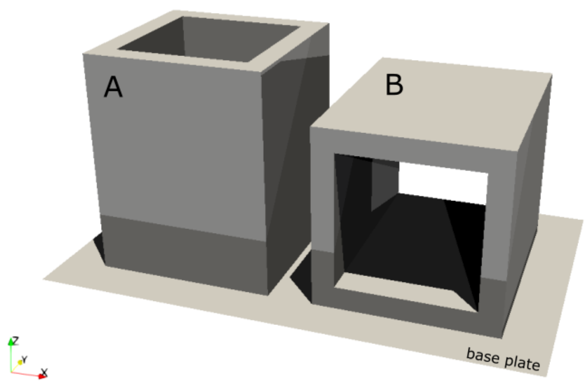

Figure 4: The same component printed under different orientations will result in different deformation pattern and residual stresses. Which configuration is better?